Properties of polygons

"Properties of polygons" program manual consists of four parts:
1. Program features.
2. Learning objectives.
3. How to use the program.
4. Problem examples.

Program features

The program shows some of the polygons' properties. Student can build his or her own figure and check its properties with functions. Especially he or she can compare the length of segments, measure the angles and also illustrate the periphery and area.

Being precise - to draw a polygon you have define consecutive points which build the figure. Next you can analyse the properties of the polygon. Particularly you can add a free point on the pane or a point attached to a segment either a point dividing the segment into half or any point of segment. Moreover for each point you can find its project on a segment. In triangle this allows constructing heights and medians.

With this application you can compare segments on three levels – it can be dependent of side's length or diagonal's length or polygon's periphery. Similarly you can compare angles on three groups, both analyse sum of angles or compare measures between angles.

After choosing the "area" option you can find out how the area changes when you resize the figure. How some measurements affect the area.

Moreover there are some questions for students with possibility to evaluation and expand.

Learning objectives

Material can be divided into three groups –triangle's properties, problems with quadrilateral and other properties of polygons.

For the first group the subjects are:
1. A rule that describes when three given lengths will make a triangle and when they will not.
2. The sum of the angles in triangle.
3. Properties for medians and heights.
4. Dependence of area on height and base.

For quadrilateral we have properties:
1. Properties of angles in different kinds of polygons.
2. Properties of diagonals for different kinds of polygons.
3. Properties for periphery and area.

For third group you can examine dependence on number of sides, vertexes and diagonals and other properties connected to area and periphery.

How to use the program

The screen is divided into tree parts:
- toolbar
- different kind of panes (POLYGON, SEGMENTS, ANGLES, AREA)
- menu panes.

Not all elements are visible at the same time – some of them you can see on demand. You can modify panes – move or change their size. If you want to close a pane click on icon like the one below:

Toolbar icons are divided into four groups:

- starting and ending of the program,
- adding a new element,
- comparing a length of segments, to compare measure of triangles or area
- other.

Starting and ending of the program

	restart the program
	quit the program

Adding a new element

Icon	Hint	Description
	insert new points - click on pane POLYGON	If you click on the pane POLYGON then you can add a new point. If there are at least three points and you click right, polygon will be closed and you will see a new segment on the screen.
	insert a new point	If you want to add a new point choose one of the option: • free points • on the segment- on any location, on the middle of the segment, on intersection of two segments • projection a point on a segment
	create a new segment - connect points	To make a new segment you must choose two different points, not connected with segment.

Especially if you want to create a new point choose:

- „free points" and click somewhere on the plane POLYGON
- "in the segment" –choose the proper segment
- "middle of the segment" – you also choose the proper segment
- „on intersection" –choose two different segments
- „projection" – you select the point you want to project, next the segment.

You can also change the colour of a segment – if you want to do this you must click on a segment with the right mouse button.

Comparing a length of segments, to compare measure of triangles or area:

Icon	Hint	Describe
	put segments	User chooses a segment by clicking on it or by clicking on two different points. Two ways to put segment are added in the program to avoid ambiguous with two segments overlap each over. Moreover you must to determine a level to put segments.
	put an angle	The same thing like for segments you can do with angels, to compare their measure. Just determine three points. The orientations of all angels will be clockwise. Also you must determine number of the angle you want to put on a new angle.
	show an area	Green rectangle illustrates area of the polygon. With slider you can change width of the rectangle, the height will fit automatically. The high of rectangle is limited by the size of pane AREA. The application doesn't show the right area, if polygon intersects with itself.

Put segments

If you want to compare two or more segment's length, you must change a level to put segments on different levels. To change a level you must click on icons like this:

In the application you can use three different levels. They change in a cycle i.e. from first level we go to second, from second to third but from third to first once more.

There is an option to delete current level. You must choose this option ("delete current level") on the button in the plane ADD SEGMENTS.

Put angles

The number of the angle you can change by clicking on an icon like this:

You can delete current angle by clicking on button "delete current angle".

Auxiliary functions:

Icon	Hint	Description
	you can freely move points	This mode let user to freely move the polygon. After choosing this mode you know that any new element will not be added and not be put. It is useful to analyse something, not to create new elements.
	show a grid	A mesh helps to observe some properties. It can work in three modes: • not visible mode • it is visible and you can drag it • it is visible and you can't drag it On all panes – POLYGON, SEGMENTS and AREA density of the grid is the same.
	check if the polygon intersects	At the begging is that program checks if the polygon intersects. After some period of time the applications checks if any of side intersect another. If it is true the polygon must be redrawn and put in last good position. You can turn off these options by click on the icon.
	translate captions into Polish	If you choose this options all captions will be translated into Polish or if they are Polish into the English language.
	answer the questions	If you choose this icon you can find exercises for students. There are three groups: • triangles properties • area of polygons • area and periphery.

Properties of triangles

There are different properties of triangle, student makes illustrations to find the answer and choose yes or no when this property is true or not. The illustration on the properties is not a proof, but helps the students to understand the problem. `

In the current version of program after choosing "questions" a one of these three questions appears:

- Do medians intersect in every triangle at one point?
- Do the heights in every triangle are the same length?
- In every triangle sum of angle is 180 degree, is that true?

If student take correct answers and do all illustrations, he or she is asked to discover "new" properties. In this way he or she can draw inspiration with ready examples and other ways he or she can to it independly with constructive attitude.

Exercise is designed to learn students know new properties of triangle.

Area of polygons

For given kind of figure student ought to draw polygon of given area. The area is shown as a grey rectangle on pane AREA. Student can modify given polygon, but he or she must remember about contractions on its shape, that is - square must be a square, rhombus must be a rhombus etc. Precisely - you can move dark-green points without any restrictions, a little lightness with some restrictions, but deep-red ones you can't move.

When it seems correctly - area is equal or very similar to given in exercise - you must push button "check". Then it is visible green rectangle with area equal to the polygon's area on pane POLYGON and message about correctness of exercise's execution or about mistake.

Area is compare with precision connected with variable `dok_pole`. Precisely error of area, counted as the absolute value of the difference of the polygon's area and green rectangle's area, must be smaller that value `dok_pole * rectangle's area.`

In the current version of application the questions are drawn from following set:

- triangle's area
- quadrilateral's area
- pentagon's area
- parallelogram's area
- area of rhombus
- trapezoid's area.

If you answer all questions, the program will load a list once more.

Exercise is designed to give student intuition about area of different polygons. Student can compare area of polygon to area of rectangle, which is close to natural intuition. Especially student can examine how area changes when we change some lengths in polygon e.g. in a triangle.

Area and periphery

In exercise you must draw the given polygon on specific area and periphery. The area is given as grey triangle on pane AREA and periphery is given as grey segment on pane SEGMENTS. Exactly error of area and periphery is count like in previous exercise for values `dok_pole` and `dok_o`.

If the answer is correct user can take next problem, on mistake there is information about error in periphery or if periphery is correct about area.

Exercise is to make intuition about periphery and area for different polygons. Student can observe that exists polygons on the same area and different periphery or the same periphery and different area.

Problem examples

For a triangle:

1. A rule that describes when three given lengths will make a triangle.
2. The sum of the angles in triangle.
3. Dependence of base and height lengths on the area.
4. The point where the medians intersects.

For quadrilateral:

1. Pointing angles with the same measure in rectangles and isosceles parallelograms.
2. Area dependence on chosen values

The triangle

Step 1

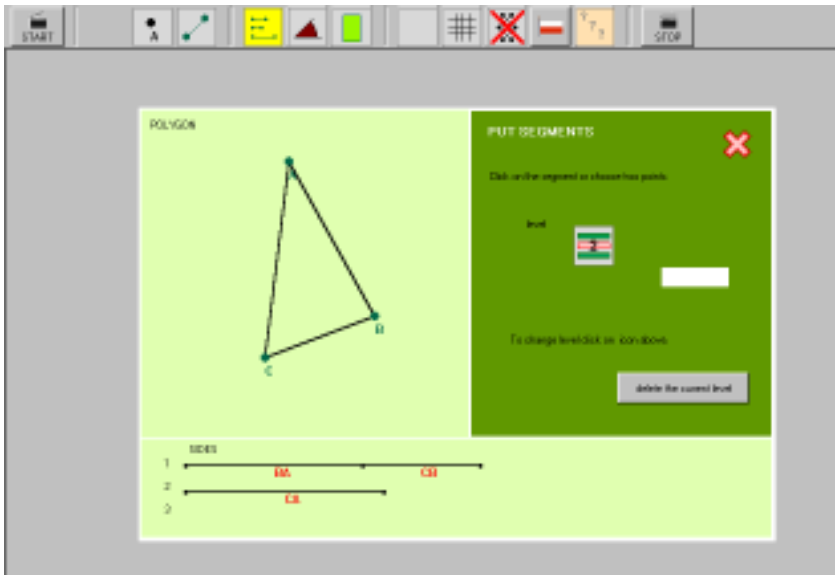
You make a triangle – click on pane POLYGON three times to make three different points and next click the right mouse button to close a polygon.

A rule that describes when three given lengths will make a triangle

Step 2

Click on the icon “add segments” and on the first level add two sides of the triangle by clicking on them. Next you take the second level and add a third side. To make sure that accidental clicks do not do not make any undesired effects, take option “you can freely move points”.

An illustration for this property is ready, ask student to draw a triangle in which sum of length for two sides will be short than length for third one. After doing some checking student ought to notice that it is impossible to do that. In this way you can inspire student to discover property.



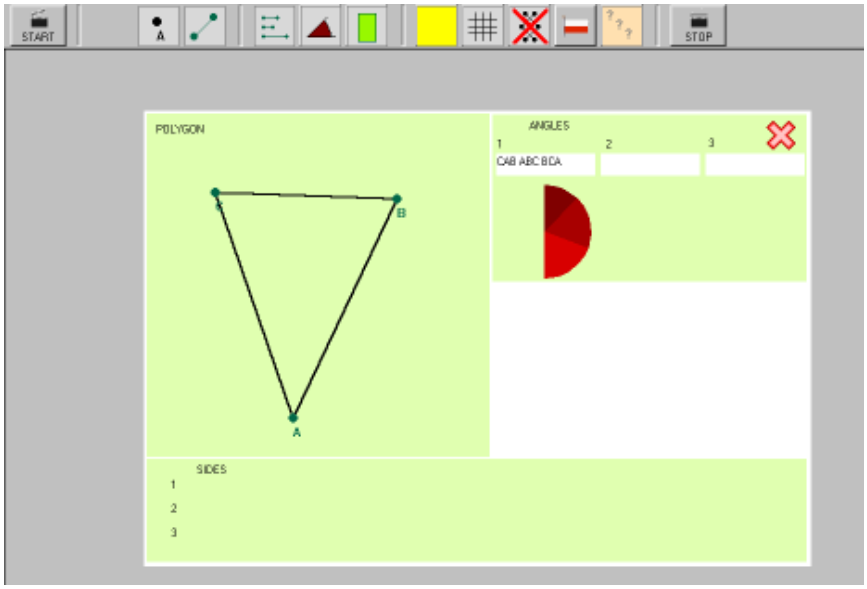
The sum of the angles in triangle.

Step 3

Click on the icon “add angle” and put three interior angles of the triangle on a same angle. To do this click three times for three vertexes. You must pay attention that an angle orientation ought to be correct.

For instance you have triangle ABC, where A – is in left bottom corner, B- is in right upper corner and C – in left upper corner. To put the first angle you choose the points C, A, B, for second angle – points A, B, C, for third B, C, A.

An illustration for this property is ready. Ask student to modify triangle and observe the sum of triangles. Student ought to notice that the sum of angles is constant and amounts 180º. Additionally you can ask student to do the same with quadrilateral.



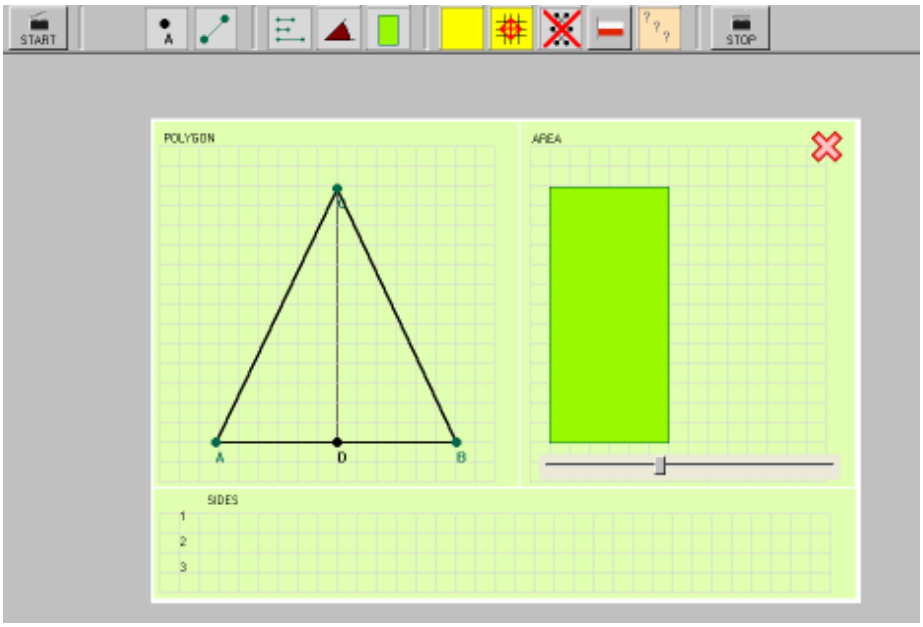
A property for area

Step 4

In this step you add a height to the triangle. First of all you make a new point – project one vertex for the side which doesn't stick to this vertex. Choose icon "add a new point" with option "project", next click on one vertex and the proper side.

For instance for the vertex C - choose the side AB. Next choose "create a new segment – connect points" and choose a new point and the vertex you have click for the first time. In our example D and C. In the last step you choose option "area" to watch how area changes.

Try to make that base horizontal and height vertical. For better observation add a grid. Everyone can notice that an area for triangle is equal to an area of a rectangle where one side is equal to a height and other is half as long as the base.



The point where the medians intersects.

Step 1 (like previous)
You have to make a triangle – click on pane POLYGON three times to make three different points and next click the right mouse button to close the polygon.

Step 2
You have to add points on the middle of all the sides. First choose the icon "insert a new point" with option "middle of the segment" and click on three different segments of the triangle. In this way, you've made points, which are exactly in the middle of the segments.

Step 3
You have to draw the middles. To be precise, you must connect points in the middle of segment with opposite vertex. Choose an icon "create a new segment" and point out the vertex and the point in the middle of the segment. In this way you've got three middles of the triangle.

Step 4

We analyze whether three middles intersect in one point or not. You choose an icon "you can freely move points". That causes the system not to generate unwanted effects after accidental mouse clicks. Next we change a position of vertex and observe if the middles intersect in one point.

The quadrilateral

Pointing angles with the same measure in rectangles and isosceles parallelograms.

Step 1

You have to make a quadrilateral – click on pane POLYGON four times to make four different points and next click the right mouse button to close the polygon.

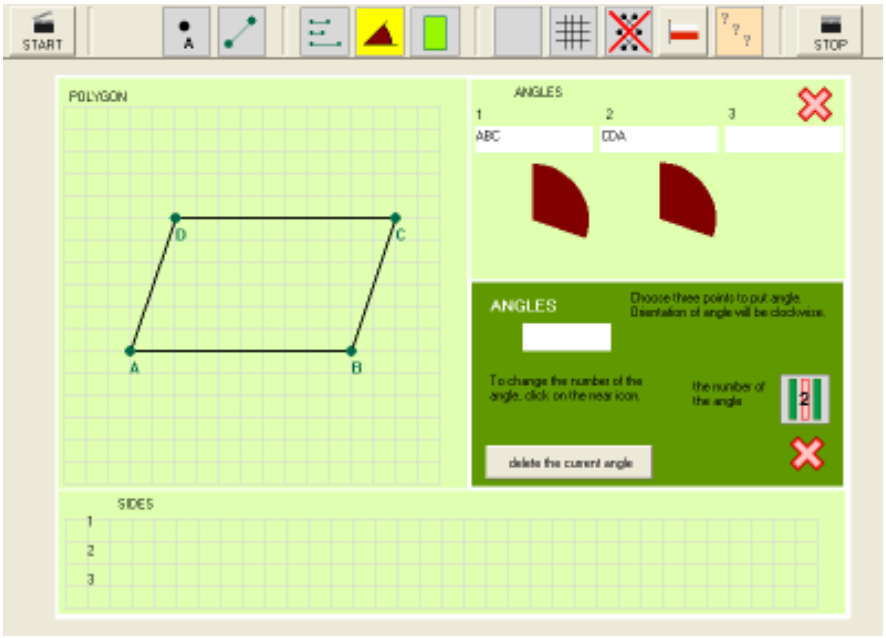
Step 2

For a quadrilateral ABCD you put angles ABC and CDA, you analyze for which angles they are the same measure.

Exactly you have to choose icon “put an angle” and select in sequence three vertexes A, B and C. Next you click on the icon ‘the number of the angle” and come to another angle. Similarly you choose three points C, D and A. In the same way you put the next angle.

Step 3

We examine for which quadrilaterals the angles are the same. You must change a position of the vertexes to get a square, a rectangle that is not a square, rhombus etc. You observe for which figures the angle is the same measure.



Area dependance on chosen values

Activity similar like for a triangle.

Module 1
Different ideas of how to apply triangle inequality.

Program:

To solve problems you can take the package “Triangle inequality”.

Activities:

1. Try to draw a triangle, where the sum of two sides is less than a third one.
   An auxiliary question: When a sum of two sides is equal to a third one?

2. Draw a triangle ABC and a point D inside the triangle. Check relations between the sum of the distance from the point D to vertexes and periphery of the triangle.
   Are the relations true for the point D which is outside a triangle?

3. In quadrilateral sum of the lengths for diagonals is more than a half periphery. Is that true or not?
   Is it less than periphery? Consider convex and concave quadrilaterals.

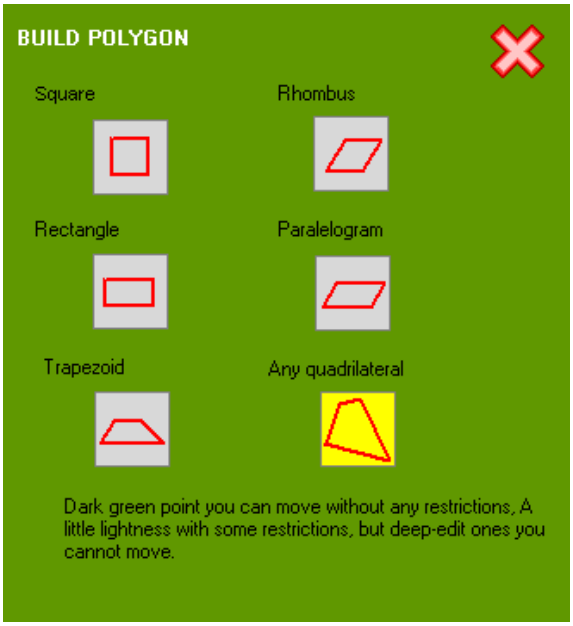
4. When in a quadrilateral ABCD the sum of the length of two following sides is is less than the sum of the length of two remaining ones. That is BA + CB < DC + DA?

5. Look for other ideas of how to apply triangle inequality.

Module 2
Quadrilaterals properties - sumary

Program:

To solve problems you can take the program "Quadrilaterals".



Activities:

1. Check properties of quadrilaterals and fill the table.

Questions in the table are as follows:

a) Are all sides equal?
b) Are all angles the same?
c) Do opposite angles are congruent?
d) Are diagonals equal?
e) Do diagonals bisect at the right angle?
f) Do diagonals bisect each other?

1. Compare you results with your neighbor, if you have some doubt check it using the program.

   Examples of questions you can add:

   a) Are at least two sides parallel?
   b) Are opposite sides the same length and parallel?
   c) Are two pair of angles are the same measure?
   d) Are all angles right?
   e) Must any angle be a right angle?
   f) Is the sum of interior angles equal to 360°?

2. Write next properties and check it for different kinds of quadrilaterals.

3. Compare your ideas with other students, if some has a new property, add it to your table.

4. Choose one property, describe the one you've chosen and prepare an image (a screen capture) below the table.

Module 3
Quadrilaterals properties - a quadrilateral in a quadrilateral

What are the properties of a quadrilateral, which vertexes are in the middle of the sides of another quadrilateral?

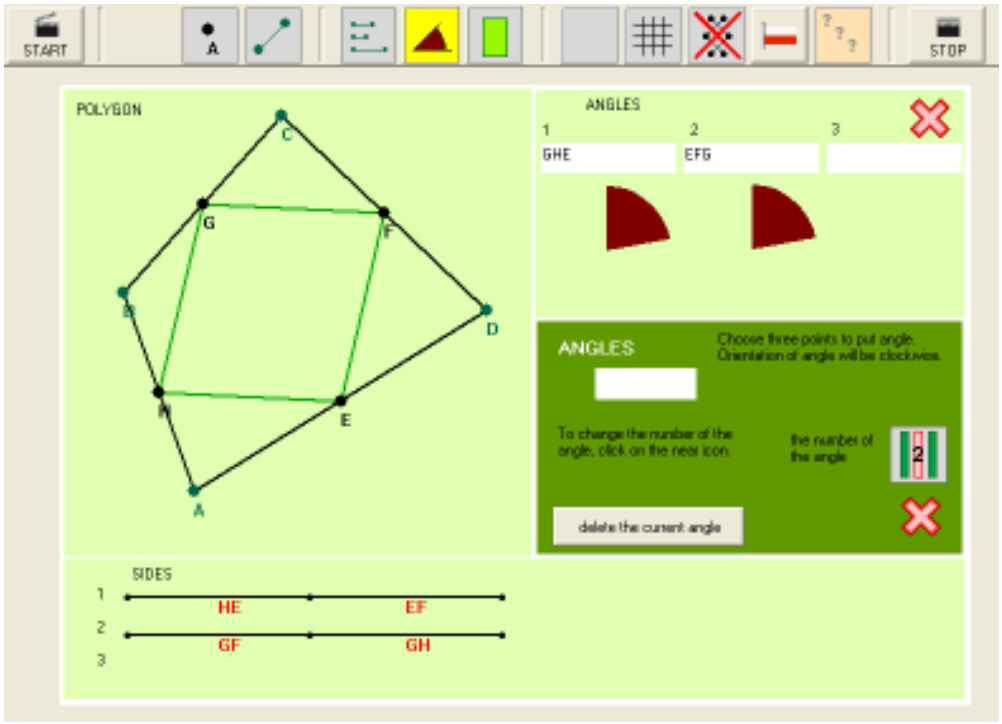
Program:

To solve problems you can use the program "Quadrilaterals".

Exercises:

1. Construct unrestricted quadrilateral ABCD. Add four points, which will be in the middle of the sides (properly EFGH). What are the quadrilateral EFGH properties?

   Auxiliary questions:
   a) Are all the sides equal?
   b) Are all the angles equal?



1. Examine, for what position of points A, B, C, D quadrilateral EFGH will be
   a) a rectangle
   b) a square.