Microworld Vectors – Exercise Book

The aim of this microworld is to introduce different ways of presenting vectors: graphic representation, analytic representation in Cartesian coordinates system, analytic by length and direction (circular coordinates system) and by means of colors (model of space of colors). The exercises show the connections between different vector representations and help to understand what is vector and how it can be represented.

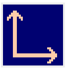
The exercises are “open”. There are possibilities of different kinds of activity, gathering the experience, answering questions: what will happen when we change...?

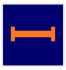
Buttons:

 Jump to the main page, project icon.

 Show / hide the axes of coordinates system.

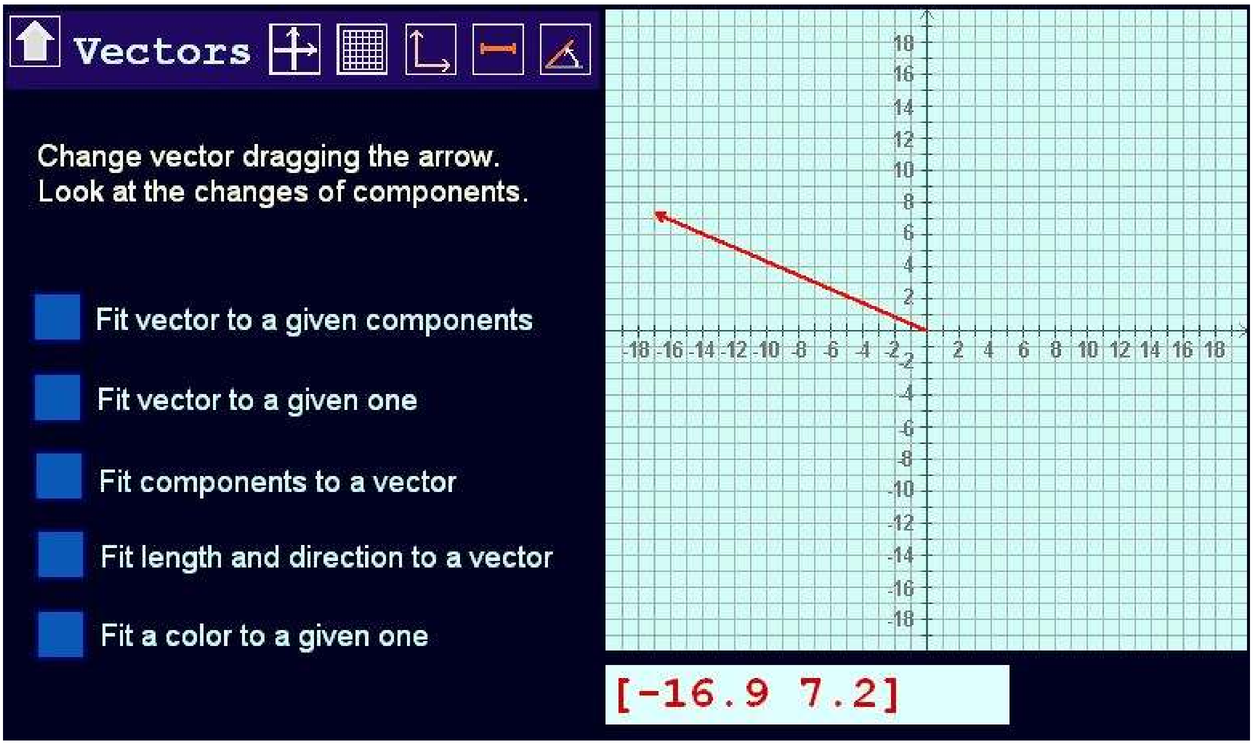
 Show / hide the grid.

 Show / hide the vector components (arrows).

 Show / hide the field with vector length.

 Show / hide the field with vector direction (in degrees).

 Jump to the next page (exercise).



<u>1. Vector and its components</u>

Short instruction

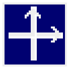
Change vector dragging the arrow. Look at the changes of components.

Model
Vectors are different from numbers. A number may be presented by a point on numbers axis. It is than connected with the distance between this point and zero on the axis. If we try to represent in a similar way a vector on a plane we should draw a coordinate system, find a point, connect the point with the centre of coordinate system and show the point by drawing an arrow (just to point out which end of segment is our point). The segment with an arrow at the end is a graphic representation of a vector. In this representation vector has:
value (length) – segment length,
direction – an angle between a segment and X axis counted anticlockwise,
sense – shown by an arrow,
spot – the centre of coordinates system.
Vector on the plane is than explicitly represented by two numbers: X coordinate and Y coordinate of the point so it may be written as a pair of numbers [x y]. Those numbers we will call vector components.
The first page of microworld shows two representations of a vector – graphic and analytic. The vector placed on the light blue pane may be changed by dragging an arrow. Changing vector arrow causes change in analytic representation shown in the field below a pane.

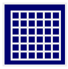
Activities
1.1. Drag vector arrow to different places of blue pane. Look at the changes of components in analytic representation. Write down your remarks.

1.2. Check how big is a blue pane. Drag vector arrow to the borders of a pane. Write down results.

1.3. Switch on the axes of coordinates system. Check results of activity 1.2. Check the order of components in analytic representation.

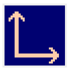


1.4. Switch on the grid on coordinates system. Using it try to set up vectors:

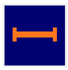


[1 1], [10 10],
[5 -8], [-5 8],
[7 7], [-7 -7],
[9 -15], [-15 9]
Describe how are vectors in pairs interconnected.

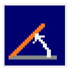
1.5. Switch on the components presentation on the plot. Find and write down 4 different vectors with the same values (lengths) of components.



1.6. Switch on the field with value (length) of vector. Find and write down 4 different vectors with the same values (lengths).

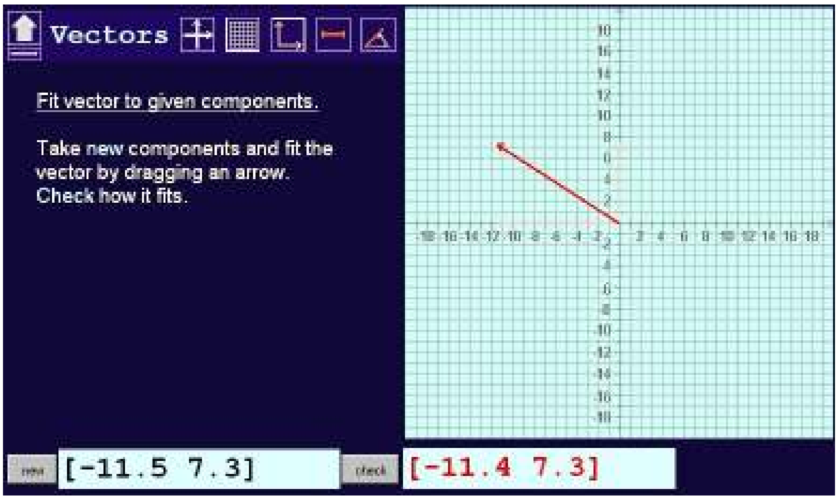


1.7. Switch on the field with the direction of vector. Find and write down 3 different vectors with the same direction.



On screen:
pane with vector,
text field with vector components (analytic vector representation),
hidden text fields with length and direction of vector,
5 buttons allowing to jump to the other pages.

2. Fit vector to given components

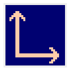


Short instruction
Press **new** button for new components. Fit to them vector by dragging the arrow. Check how it fits (press **check** button).

Model
Having a graphic representation of vector you may easily fix vector components on the axes of coordinates system. Is it also easy to draw vector having its components? Guess, yes. You should only remember the order of components in analytic representation. The first is X component (horizontal), the second Y component (vertical). It lasts only to find a point with coordinates X and Y and place there the end of vector arrow.
Using **new** button you may pick at random analytic representation of new vector which is placed in the field on the left. Now by dragging a red vector arrow on a blue pane you should fit graphic representation of vector to its analytic representation. Pressing **check** button allows to check how it fits. The analytic form of red vector will be shown. If the difference in both components is not greater than 1 it may be assumed that the fit is good. The precision of fitting is commented on dark blue pane.

Activities
2.1. Fit vector arrow to given components in its analytic representation. Do it few times. Pick up new components (**new** button). Dragging red vector arrow try to archive the best fit to given components. Than check how it fits (**check** button). You can see red components of a drawn (fitted by you) vector. Compare components. If the difference in both components is not greater than 0.2 you may assume that the fit is excellent. If the difference is not greater than 1, the fit is still pretty good.

2.2. Switch on the components presentation on the plot. Do once again previous exercise. Is it now going better.



On screen:
pane with red vector (to adjust by dragging the arrow),
text field with vector components (black numbers),
button **new** – picking the new components to fit the vector (arrow) to them,
button **check** – checking red vector components (showing text field with this components),
hidden text field with vector components (red numbers),
hidden text fields with length and direction of vector.

3. Fit vector to a given one

Short instruction

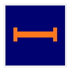
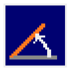
Pick at random a new vector (green) and try to fit to it the red vector on the left pane (by dragging the arrow). Check how it fits (press **check** button).

Model

Comparing analytic forms of vectors seems to be simple. Two vectors are identical having the same components. Is it equally simple to compare two graphic forms of vectors? You should compare lengths, directions and senses of vectors. Two vectors are identical having the same lengths, directions and senses. Remark: often it is assumed that a direction of a vector is a direction of line on which vector lies. Than you may say for example that two vectors have the same lengths, directions and reverse senses, but those vectors are not equal.
Using **new** button you may pick at random new green vector placed on light blue pane on the left. Now by dragging a red vector arrow on a light blue pane on the right you should fit it to the shape of green vector. All the time you can see analytic form of red vector. Than you may check how vectors are fitted using **check** button. The analytic form of green vector will be displayed. If both components of vectors are similar (the difference in both components is not greater than 1), than you may assume that the fit is good.

Activities

3.1. Fit red vector to given green one. Check how it fits (**check** button). The components of green vector will be displayed. The precision of fitting will be commented on light blue pane. Repeat the exercise few times. Pick up new green vector (**new** button) and fit to it red vector. If its not fitted good try to improve it.

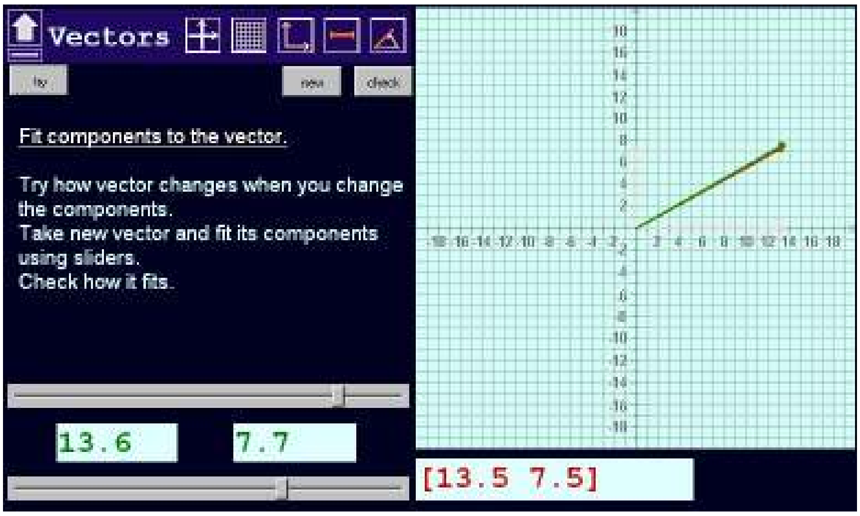



3.2. Switch on the fields with length and direction of vector. Repeat previous exercise comparing length and direction of a red vector set by you with length and direction of picked green vector. If its not fitted good try to improve it. In which case it was easier to fit vectors.

On screen:

pane with red vector (to adjust by dragging the arrow),
pane with green vector (on the left side),
button **new** – picking the new green vector,
button **check** – showing green vector components, its length and direction,
text field with red vector components (red numbers),
hidden text field with green vector components (green numbers),
hidden text fields with length and direction of red vector,
hidden text fields with length and direction of green vector.

4. Fit components to the vector.



Short instruction

Try how vector (arrow) changes when you change the components (press **try** button and use the sliders). Take new vector (press **new** button) and fit its components using sliders. Check how it fits (press **check** button).

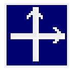
Model

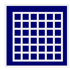
How the graphic form of vector is changing when we change components. Of course vector is changed. If we change any component than length and direction of vector are changed (if only vector is not laying on axes).

Activities

4.1. Try how graphic representation of vector is changing when you change its components. First change X component using upper slider. Than change Y component using lower slider. Look how green vector is changing. Describe how it is changing when you change X and Y component.

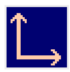
4.2. Switch on the axes of coordinates system and a grid. Set up integer value of one component and change the other. If its necessary improve your description in previous exercise.





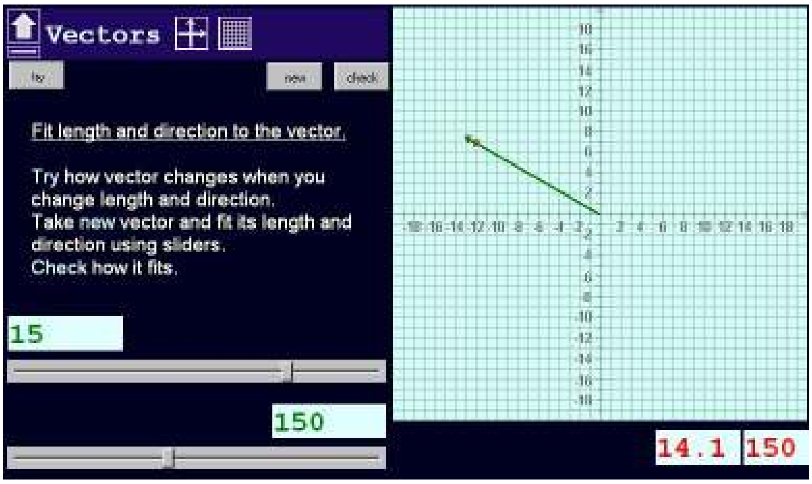
4.3. Pick up new red vector (using **new** button). Set on sliders its components. Check how they are fitted (**check** button). The green vector with set coordinates will be displayed and the components of picked red vector will be shown. If necessary correct green vector using sliders.

4.4. Switch on the components presentation on the plot. Do once again previous exercise taking into account visible lengths of components.



On screen:

pane with green vector (to adjust by the sliders),
slider to set X component of the green vector,
text fields with X and Y components of a green vector,
slider to set Y component of the green vector,
button **try** – allowing to adjust green vector using sliders,
button **new** – picking the new red vector (arrow) to set its components using sliders,
button **check** – showing the green vector (arrow) and field with red vector components.



5. Fit length and direction to the vector

Short instruction

Try how vector (arrow) changes when you change the length and direction (press **try** button and use the sliders). Take new vector (press **new** button) and fit its length and direction using sliders. Check how it fits (press **check** button).

Model

How the graphic form of vector is changing when we change vectors length or direction. Of course whole vector is changing. Lets mark r vector's length and α vector's direction (an angle between X axis and vector). Than components of vector may be calculated: $x = r \cos \alpha$, $y = r \sin \alpha$, what means that we have full description of a vector. So length and direction of vector define vector in similar way like its X and Y components.
The angle which is a measure of direction is given in degrees.

Activities

5.1. Try how vector changes when you change its length and direction. First change length using upper slider. Than change direction using lower slider. Look how green vector is changing. Describe how it is changing when you change length and how when you change direction.

5.2.
Using sliders set vectors of given lengths and directions, write down how vectors are directed. Write down analytic form of vectors.
10, 0
10, 45
10, 90
10, 180
10, 270

5.3. Pick up new red vector (using **new** button). Set on sliders its length and direction. Check how they are fitted (**check** button). The green vector with set length and direction will be displayed and the length and direction of picked red vector will be shown. If necessary correct green vector using sliders.

On screen:
pane with green vector (to adjust by the sliders),
slider to set the length of the green vector,
text fields with the length and direction of a green vector,
slider to set the direction of the green vector,
button **try** – allowing to adjust green vector using sliders,
button **new** – picking the new red vector (arrow) to set its length and direction using sliders,
button **check** – showing the green vector (arrow) and field with red vector length and direction.

6. Fit a color to a given one

Short instruction

Set square color to the background color using basic colors sliders. Look at the numbers under the square. Check how the color fits (press **ready** button). Pick the new background color (press **go** button).

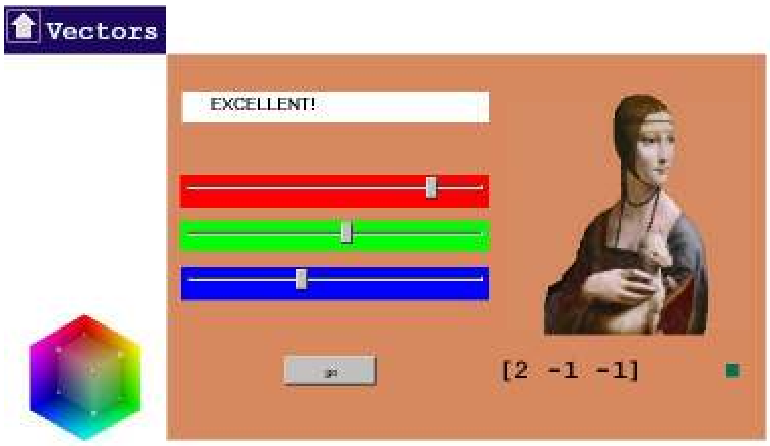
Model

Any color is a mixture of three fundamental colors: red, green and blue (RGB model).
Let assume that each fundamental color may have 256 saturation levels from 0 (no saturation) to 255 (full saturation). So color may be represented by vector with 3 components showing saturation of red, green and blue respectively. For example vector [0 255 0] will represent green. How many colors can be represented in such a way? If there are just 256 levels of saturation for each of 3 fundamental colors, so there are 256*256*256 = 16777216 (near to 17 million) different possibilities. This way of representing colors is very useful in computer graphics. There are a lot of colors. Our eyes may have problem with differentiation of colors which components are similar (but not equal). Lets check what is your ability to distinguish similar colors.

Activities

6.1. Using sliders change the color of the square placed on the right and compare the color and color vector form.

6.2. Using sliders set given color vectors. Write down the colors.
[255 0 0]
[0 255 0]
[0 0 255]
[255 255 0]
[255 0 255]
[0 255 255]
[255 255 255]
[0 0 0]



6.3. Check what are the colors of given vectors with equal components:
[200 200 200], [150 150 150], [100 100 100] i [50 50 50]

6.4. Press **go** button. New pane color will be picked at random. Using sliders change the color of a square to fit pane color. Check how it fits pressing **ready** button.

On screen:

pane with a background color picked at random and on it:
- text field with instruction,
- 3 sliders to set basic colors intensity: red, green and blue,
- button **go** / **ready** picking new background color (**go**) and checking how it is fitted (**ready**),
- square pane to set color using sliders,
- text field with color vector (basic colors intensity)
- button – green square – showing a picture on the square pane.